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Rising Concentration and Wage Inequality*

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Abstract

Wage inequality has risen in many countries over recent decades. At the same time, production has become increasingly concentrated in "superstar" firms. In this paper, we show that these two phenomena are linked. Theoretically, we show that shocks that increase concentration, such as an increase in consumers' price sensitivity, will also lead to an increase in wage dispersion between firms. Empirically, we use industry-level data from 14 European countries over the period 1999–2016 and show robust evidence of a positive and statistically significant correlation between concentration and the dispersion of firm-level wages.

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1 Introduction

In recent years, two important economic phenomena have received a large amount of attention from academics and policymakers. On the one hand, there has been a strong increase in wage inequality since the 1980s (Juhn et al., 1993; Katz and Autor, 1999; Acemoglu and Autor, 2011). A recent literature has shown that a large fraction of this increase in inequality is driven by increased wage dispersion between firms, rather than within firms (Card et al., 2013; Song et al., 2018; Barth et al., 2016). On the other hand, a separate literature has shown that product markets have become increasingly concentrated, with a smaller number of firms becoming increasingly dominant in many industries (De Loecker and Eeckhout, 2020; Autor et al., 2017, 2019; Grullon et al., 2019; Kehrig and Vincent, 2018; Azar et al., 2017, 2018; Benmelech et al., 2018). So far, the rise in inequality and the rise in concentration have been studied in isolation. In this paper, we show that the two phenomena are related.

From a theoretical perspective, we show that a shock that increases concentration, such as an increase in consumer price sensitivity (which in turn may be driven by factors such as greater economic integration and the availability of new web technologies), will also lead to increased wage dispersion between firms. Empirically, we use industry-level data from 14 European countries over the period 1999–2016 to show that there is a significant positive correlation between inequality and concentration, which is robust to controlling for unobservable factors in a variety of ways.

We motivate our analysis using the heterogeneous firm search and bargaining framework of Helpman et al. (2010). We analyze the implications of an increase in consumer price sensitivity – modeled as an increase in the elasticity of substitution between varieties in consumption, as in Autor et al. (2019).[1] The model predicts that this type of shock will lead to increased concentration of production in the most productive firms within industries, while at the same time increasing wage inequality between firms within industries.

Intuitively, an increase in price sensitivity shifts consumer demand towards the most productive firms, who are able to produce goods at a lower price. Low productivity firms are no longer able to operate profitably and must exit the market, while high productivity firms are able to increase their market share. These highly productive firms will increase both the quantity and the quality of the workers that they hire,

[1] The conceptual framework considered by Autor et al. (2019) features a competitive labor market, which does not allow for any type of wage inequality.

hence increasing between-firm wage inequality.

The link between concentration and wage inequality carries over to models featuring other mechanisms that link firm performance to worker wages. For example, in the setting of Egger and Kreickemeier (2012), worker wages are proportional to firms' operating profits due to fair-wage considerations. A shift in demand towards the more productive firms will increase their size and profitability, as well as the wages of the workers that they hire, hence leading to increased concentration and increased between-firm wage inequality.

We test the predictions of the model regarding the link between concentration and inequality using data from the Competitiveness Research Network (CompNet). The dataset provides information on concentration and wage inequality at the 2-digit industry level for 14 European countries over the period 1999-2016. Concentration can be measured in terms of the sales or employment shares of the top firms within an industry-country-year cell. Wage inequality captures differences in average labor costs per worker across firms.

Our empirical strategy consists of regressing inequality on concentration as well as various combinations of industry, country, and year fixed effects. This strategy allows us to control for different types of confounding factors, and to exploit different sources of variation for identification in order to determine the robustness of the results. Our key finding is that there is a positive and statistically significant correlation between concentration and inequality, which is robust to various ways of restricting identification – including, notably, when we exploit only variation within industry-country cells over time. This suggests that, as predicted by the model, the two phenomena are indeed linked to each other.

When we explore changes at different parts of the firm wage distribution, we find that concentration is correlated with wage decreases at the lowest part of the distribution, and wage increases at the top of the distribution. The relationship is non-linear, with particularly strong wage increases at the 99th percentile. This suggests that the very top firms are important in driving the increase in between-firm wage inequality – though inequality is also higher in more concentrated industries, even if the firms in the top 1% are excluded. We also find that the positive correlation between concentration and inequality is observed within most countries in our sample, and within almost all sectors of the economy.

Our paper provides an important contribution to the literature by providing evidence of the link between the growth of concentration and the growth of wage inequality.

Our results highlight the importance of considering common driving forces that can account for both of these patterns. We contribute to the literature that studies changes in wage inequality by proposing a mechanism which gives a relevant role to firms in driving the widening of the wage distribution. While several papers have documented the increase in wage dispersion between firms, the literature on wage inequality has mostly focused on the role of changing demand for skills and tasks, without allowing for heterogeneous firms to play a relevant mediating role (see e.g. Acemoglu and Autor, 2011, for a review of this literature). Studying the specific nature of the shocks that favor the most productive firms in an industry remains a promising avenue for future research on the drivers of wage inequality.

2 Theoretical Motivation

In order to motivate our analysis of the relationship between concentration and inequality, we illustrate the theoretical link between these two variables within the closed-economy framework of Helpman et al. (2010). Their model introduces Diamond–Mortensen–Pissarides (Diamond, 1982a,b; Mortensen and Pissarides, 1994) search and matching frictions into a Melitz (2003) model with heterogeneous firms. The model is able to generate wage differences between firms through a combination of: (i) search frictions and wage bargaining, and (ii) heterogeneous match-specific ability and the availability of a screening technology.[2] We refer the reader to the Helpman et al. (2010) paper for full details on their model. Here, we focus on the implications of their equilibrium conditions for concentration and between-firm wage inequality. In particular, we focus on the implications of an increase in consumer price sensitivity, modeled as an increase in the price elasticity of demand, as in Autor et al. (2019). Autor et al. (2019) discuss how consumers may have become more price-sensitive due to greater product market competition (e.g., through globalization) or new technologies (e.g., due to greater availability of price comparisons on the Internet). While Autor et al. (2019) consider the implications of this type of shock within the setting of a competitive labor

[2]Note that in this model, match-specific ability and screening are crucial ingredients to generate wage variation across firms. Search frictions and wage bargaining alone are not sufficient to generate between-firm wage heterogeneity (see for instance Felbermayr et al. (2011)). The intuition is that, in a standard search and bargaining model with heterogeneous firms, the additional value created by the marginal worker is identical across firms in the presence of a common search cost. Thus, more productive firms will be larger, but wages will be equated across firms with different productivity levels.

market (with no wage inequality), we extend their analysis in order to consider the implications within the context of a framework that allows for wage inequality.

2.1 Key Features of the Helpman et al. (2010) Model

As in Melitz (2003), each sector features a continuum of horizontally differentiated varieties, with total consumption Q being given by a constant elasticity of substitution (CES) aggregate:

$$Q = \left[\int_{j \in J} q(j)^{\beta} dj \right]^{1/\beta},$$

where j indexes varieties, J is the set of varieties within the sector, $q(j)$ denotes consumption of variety j and $\beta \in (0, 1)$ is a function of the elasticity of substitution between varieties, σ, namely $\beta \equiv (\sigma - 1)/\sigma$. The product market is characterized by a continuum of monopolistically competitive firms, each producing a unique variety and facing a fixed cost of production. Firm output is given by:

$$y = \theta h^{\gamma} \bar{a}, \qquad 0 < \gamma < 1$$

where θ is the firm's idiosyncratic productivity draw, h is the measure of workers hired and $\bar{a}$ denotes the average match-specific ability of these workers. The productivity distribution, $G(\theta)$, is assumed to be Pareto with shape parameter z.[3]

Workers are ex-ante identical but differ in terms of their match-specific ability, which is not transferable across firms. In the Appendix, we discuss an extension of Helpman et al. (2010) to two types of workers that allows us to relate increased concentration and wage inequality to changes in worker sorting on observables. We discuss this extension in further detail below, but for now we focus on the setting with ex-ante homogenous workers. Workers' match-specific ability is drawn from a Pareto distribution, $G_a(a) = 1 - (a_{min}/a)^k$. This ability is not directly observable when a firm and a worker match, but firms have access to a screening technology. In particular, by paying a screening cost of ca_c^{δ}/δ, a firm can identify workers with an ability threshold below a_c. Neither the firm nor the workers know the match-specific abilities of individual workers, so bargaining occurs under conditions of symmetric information.

[3] The assumption of a Pareto distribution is common in the literature on heterogeneous firms. Corcos et al. (2012) find empirical support for this assumption; Axtell (2001) shows that the observed distribution of firm sizes follows a Pareto distribution.

In equilibrium, more productive firms have incentives to screen more intensively, due to the complementarity between workers' abilities and firm productivity. Hence, more productive firms will hire workers with higher match-specific abilities and will pay higher wages. Intuitively, firms are able to adjust their bargained wage down to the replacement cost of a worker. Since more productive firms hire workers of higher average ability, their workers are costlier to replace, and hence are paid a higher wage. From the perspective of the worker, the expected wage conditional on being sampled is the same across all firms. Under the assumption that $\delta > k$, more productive firms will also be larger.

Equilibrium firm-level revenues, employment and wages are given by:

$$r(\theta) = r_d \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\Gamma}}, \qquad r_d \equiv \frac{1+\beta\gamma}{\Gamma} f_d$$

$$h(\theta) = h_d \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\Gamma}(1-k/\delta)}, \qquad h_d \equiv \frac{\beta\gamma}{\Gamma}\frac{f_d}{b}\left[\frac{\beta(1-\gamma k)}{\Gamma}\frac{f_d}{c a_{min}^{\delta}}\right]^{-k/\delta}$$

$$w(\theta) = w_d \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta k}{\delta\Gamma}}, \qquad w_d \equiv b\left[\frac{\beta(1-\gamma k)}{\Gamma}\frac{f_d}{c a_{min}^{\delta}}\right]^{k/\delta}$$

where θ_d is the equilibrium productivity threshold. As is standard in heterogeneous firm frameworks (such as Melitz (2003)), only firms with idiosyncratic productivity draws above the threshold θ_d will choose to remain in operation. b is the search cost, f_d is the fixed cost of production, and $\Gamma \equiv 1 - \beta\gamma - \frac{\beta}{\delta}(1-\gamma k)$, where, as in Helpman et al. (2010), it is assumed that $0 < \gamma k < 1$ so that firms have an incentive to screen.

2.2 Concentration and Wage Inequality

In order to measure concentration, consider the set of firms in the top μ% of the productivity distribution (among operating firms).[4] The equilibrium relationships described above imply that the share of aggregate sectoral revenues accruing to these firms, and the share of aggregate employment concentrated in these firms is given, respectively,

[4]Derivation details of the concentration measures and wage distribution can be found in Appendix A.

by:

$$C_r = \mu^{1-\frac{\beta}{\Gamma z}} \qquad\qquad C_\ell = \mu^{1-\frac{\beta}{\Gamma z}(1-k/\delta)}. \tag{1}$$

Regarding inequality, we are interested in the distribution of wages across firms. The equation for equilibrium wages implies that the distribution of wages across firms is given by:

$$G_f(w) = 1 - \left(\frac{w_d}{w}\right)^{\frac{\delta\Gamma z}{\beta k}}$$

This is a Pareto distribution with scale parameter w_d and shape parameter $\frac{\delta\Gamma z}{\beta k}$. Scale-invariant measures of inequality, such as the coefficient of variation, the Gini coefficient, or the Theil index, are decreasing in the shape parameter and are independent of the scale parameter.

2.3 Effects of an Increase in Consumer Price Sensitivity

As mentioned, we consider the impact of an increase in consumer price sensitivity, modeled as an increase in the elasticity of substitution between varieties, σ (as in Autor et al., 2019). Our two key predictions are the following:

Prediction 1: An increase in consumer price sensitivity increases concentration in terms of revenues and in terms of employment.

Proof: *Given the definitions of β and Γ and since $\gamma k \in (0,1)$, we have that:*

$$\frac{\partial \beta}{\partial \sigma} = \frac{1}{\sigma^2} > 0 \qquad \text{and} \qquad \frac{\partial \Gamma}{\partial \sigma} = -\frac{\partial \beta}{\partial \sigma}\left[\gamma + \frac{1}{\delta}(1-\gamma k)\right] < 0.$$

It is then straightforward to show that:

$$\frac{\partial C_r}{\partial \sigma} > 0 \qquad \text{and} \qquad \frac{\partial C_\ell}{\partial \sigma} > 0.$$

Prediction 2: An increase in consumer price sensitivity increases inequality in firm-level wages.

Proof: *Recall that the shape parameter of the distribution of firm-level wages is $s \equiv \frac{\delta\Gamma z}{\beta k}$.*

Given the definitions of β and Γ, is is straightforward to show that:

$$\frac{\partial s}{\partial \sigma} < 0.$$

A decrease in the shape parameter will unambiguously increase any scale-invariant measure of inequality.

2.4 Intuition

Intuitively, a shock that increases consumers' price-sensitivity will shift consumer demand towards the lower cost varieties produced by higher-productivity firms. As demand for the higher cost varieties produced by low productivity firms falls, they are no longer able to operate profitably and must exit. This leads to an increase in the productivity threshold θ_d. It is worth noting that, although the increase in the threshold reduces the range of firm types that operate, under the assumption that productivity is Pareto-distributed, this will actually *increase* the variance of productivity among operating firms.[5] Since employment and wages are proportional to productivity, the exit of unproductive firms will increase the measured dispersion in employment and wage outcomes across firms, even if there are no changes in the level of employment and wages among continuing firms.

This increased dispersion due to the selection margin is compounded by the changes in employment and wage choices among continuing firms. To meet increased demand, high-productivity firms increase their output by hiring more workers and screening more intensively, which leads to an increase in both employment and wages relative to less productive firms. As a result, sectoral concentration and wage inequality increase. Overall, the model highlights three channels through which wage inequality increases: selection, differential firm growth and differential wage growth. By shifting employment towards the most productive firms, the second channel leads to an increase in wage inequality even if wages within each firm remained fixed. Since wages increase relatively

[5]Note that the productivity distribution among operating firms is a truncated version of $G(\theta)$. This truncated Pareto has the same shape parameter z, but has a scale parameter θ_d. The variance of a variable with a Pareto distribution is increasing in the scale parameter and decreasing in the shape parameter, so as θ_d increases, the variance of θ among operating firms increases as well. Intuitively, the increased selection due to the increase in the threshold θ_d increases heterogeneity across firms as measured by the variance of productivity, due to the exit of a mass of firms with relatively homogenous firm types, which implies a relative increase in the mass of firms towards the tail. Detailed derivations can be found in Appendix B.

more at the top of the productivity distribution, the last channel further exacerbates the selection and differential firm growth effects in increasing wage inequality.

Note that the implications of an increase in consumer price sensitivity for concentration and wage inequality carry over to other heterogeneous firm settings that feature a mechanism that links worker wages to firm rents. For example, in the fair-wage framework of Egger and Kreickemeier (2012), workers adjust their effort according to whether they perceive the wage that they receive to be fair. The "fair-wage" is anchored by a firm-external point of reference (workers should consider their wage to be fair relative to the wage of employees at other firms), as well as a firm-internal point of reference (workers should consider their wage to be fair given their own firm's performance). In equilibrium, it is in the firms' best interest to pay workers the fair wage in order to elicit the optimal amount of effort.

In this framework, an increase in consumer price sensitivity also leads to a relative increase in the demand for the varieties produced by the most productive firms. As in the Helpman et al. (2010) model, this will lead to changes in selection through an increase in the operating productivity threshold, and to increases in the relative size of the most productive firms in terms of employment and sales (increased concentration). The increased profitability of the most productive firms translates into higher relative wages for workers in these firms, due to the fair-wage considerations. Hence, in this model, increases in concentration and increases in (between-firm) wage inequality are also linked.

2.5 Accounting for Sorting on Observables

The empirical literature has shown that there is an important role for changes in worker sorting in accounting for the increase in between-firm wage inequality in several countries (Card et al., 2013; Song et al., 2018).[6] There is also evidence that increased outsourcing opportunities have led to increased establishment specialization and worker segregation (Cortes and Salvatori, 2019).

The model discussed so far only predicts increases in wage dispersion among ex-ante homogeneous workers, without allowing for these empirically-relevant changes in sorting patterns. In Appendix C, we consider the Helpman et al. (2010) extension that features two types of workers (skilled and unskilled). We show that, in this setting, an increase in consumer price sensitivity also leads to increases in both concentration and

[6]See also Akerman (2019).

between-firm wage inequality. The increase in between-firm wage inequality in this case is driven both by: (i) increased sorting of skilled workers to the most productive firms, and (ii) increases in between-firm wage inequality conditional on (observable) worker skill type. These predictions are consistent with the observed increases in between-firm wage inequality documented in the empirical literature.

3 Data

In order to test the prediction of the model about the link between concentration and inequality, we use data from the Competitiveness Research Network (CompNet). This dataset draws on various administrative and public sources, and compiles information for non-financial corporations with at least one employee in various European countries. The latest edition of the data (6th vintage) provides information for 18 countries over the period 1999–2016, though not all years are available for all countries. We focus on the 14 countries which have representative data for the full universe of firms.[7] The data is made available at various levels of aggregation. We work with the finest level of aggregation available in the public-use data, which is the industry-country-year level, where industries are coded at the 2-digit NACE Revision 2 level.

A number of concentration measures can be constructed from the CompNet data. We focus on three measures of concentration in terms of sectoral sales (turnover): the share of sales in the top 1% of firms in each industry-country-year cell, the share of sales in the top 10 firms, and the Herfindahl-Hirschman index of market concentration. We also consider a measure of *employment* concentration based on the relative size of the largest 1% of firms within an industry-country-year cell.[8]

Our measures of sectoral wage inequality are also constructed from the CompNet data. Information is available on the distribution of labor costs per employee across

[7]The 14 countries are Belgium, Croatia, Denmark, Finland, France, Hungary, Italy, Lithuania, Netherlands, Portugal, Romania, Slovenia, Spain and Sweden. Due to missing data on some variables, Denmark, Netherlands and Romania are not included in all specifications. Representativeness is achieved through a reweighting procedure as detailed in the CompNet User Guide available at `https://www.comp-net.org/fileadmin/_compnet/user_upload/Documents/User_Guide_6th_Vintage.pdf`. Although data for the Czech Republic is available, its use is not recommended due to the very low coverage rate of small firms.

[8]More formally, what is available in the dataset is the volume of sales (employment) for the firm at the 99th percentile of the sales (employment) distribution within each industry-country-year cell. Since we do not observe sales (employment) for each of the firms above the 99th percentile of the sales (employment) distribution, we assume that the top 1% of firms all have the same volume of sales (employment). This implies that our concentration measure is a lower bound of the market share of

firms. This allows us to measure dispersion in average firm-level wages. It is worth noting that these dispersion measures will capture both pure firm wage premia (i.e. wage differences for otherwise identical workers), as well as sorting of workers to firms based on observable or unobservable characteristics.[9]

Figure 1 displays the overall evolution of concentration and wage inequality across the European countries in our sample.[10] The solid line in the top panel displays the evolution of between-firm wage inequality, measured as the weighted average of the 90-10 ratio of firm-level wages in each country-industry cell (where each cell is weighted based on its share of national value added). The dashed line displays the evolution of concentration, measured as the average share of revenues concentrated in the top 1% of firms within each country-industry cell. Both series display an upward trend over our sample period, confirming that both between-firm inequality and concentration have been on the rise in Europe since the early 2000s.

In order to rule out the possibility that these patterns are driven by changing sample composition over time (given that not all countries are observed in all periods, as we show below in Table 1), we run separate regressions of inequality and concentration on a full set of country-industry dummies, and compute the average residuals from these regressions in each year. These average residuals will only vary due to changes in concentration and wage inequality within country-industry cells over time. These are plotted in the bottom panel of Figure 1, and confirm that the increases in inequality and concentration observed in the raw data are not driven by changes in the composition of countries or industries in our sample.

Appendix Figure A.1 shows that concentration in terms of employment has also been on the rise in Europe, though the magnitude of the increase in employment concentration is smaller than for concentration in terms of sales.

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the top 1% of firms. Specifically, we compute the market share of the top 1% of firms as follows:

$$\frac{s_{99th} \cdot [N_{>99}]}{s} = \frac{s_{99th} \cdot [1\% \cdot N]}{\overline{s} \cdot N} = \frac{s_{99th} \cdot 1\%}{\overline{s}},$$

where s_{99th} is the value of sales (employment) for the firm at the 99th percentile of the distribution, s is aggregate sales (employment), $N_{>99}$ is the number of firms within the top 1% of the distribution, N is the total number of operating firms and $\overline{s}$ represents the average sales (employment) in the industry. s_{99th} and $\overline{s}$ are both observed in the data.

[9] Our data do not allow us to distinguish between these two components. As discussed above, the extension of the model presented in the Appendix allows for both of these factors to drive between-firm wage inequality in terms of levels and changes.

[10] Due to missing data, Denmark and Netherlands are excluded from these graphs, but they are included in regressions that use alternative measures of inequality and/or concentration.

Panel A of Table 1 presents descriptive statistics on the evolution of concentration and wage inequality for each of the countries in our sample. Columns (1) and (2) indicate the earliest and latest year in which each country is observed, while the remaining columns display the levels and the changes in inequality and concentration observed in each country. As before, inequality is measured as the 90-10 ratio of firm-level wages in each country-industry cell, while concentration is based on the share of revenues concentrated in the top 1% of firms in a country-industry cell. National averages are constructed based on each industry's share of national value added in the respective year. Column (5) shows that the increase in between-firm wage inequality has been widespread across European countries. All countries except Finland, Romania and Slovenia experience an increase in the 90-10 ratio of firm-level wages. Meanwhile, Column (8) indicates that concentration has also been on the rise in most countries. The share of sales captured by the top 1% of firms in each industry increases over our sample period in all countries except Belgium, France and Romania.

Panel B of Table 1 presents descriptive statistics on the evolution of concentration and wage inequality across nine broad sectors.[11] Column (5) shows that the increase in between-firm wage inequality has been widespread across broad sectors in Europe. Meanwhile, Column (8) indicates that concentration has also been on the rise in most sectors. The share of sales captured by the top 1% of firms in each industry experiences particularly large increases between 2004 and 2014 in the Information and Communication sector. The Manufacturing and the Wholesale and Retail Trade sectors also experience fairly strong increases, at around 0.9 percentage points. In what follows, we confirm whether these coinciding trends observed at the national and at the sectoral level are also observed when focusing on variation within country-industry cells, while accounting for various types of potential shocks through the use of different combinations of fixed effects.

[11]For this panel, we restrict the sample to the set of countries which have consistent information for all sectors over the 2004-2014 period, which are Belgium, Croatia, Finland, France, Hungary, Italy and Lithuania, except for the Real Estate sector which excludes Finland. In order to compute concentration and inequality at the level of these broad sectors, we first compute sector-level measures for each country, by averaging across 2-digit industries using industry shares of value added as weights. We then compute an average across the seven countries in each year, giving equal weight to each country.

4 Findings

In order to explore the empirical link between concentration and between-firm wage inequality, we exploit variation across industry-country-year cells in the CompNet data. Our equation of interest is:

$$INEQ_{ict} = \alpha CONC_{ict} + \gamma_i + \delta_c + \tau_t + u_{ict} \qquad (2)$$

The dependent variable is a measure of inequality in firm-level wages in industry i in country c at time t. The key independent variable is a measure of concentration in industry i in country c at time t. In order to exploit different sources of variation for identification, and to control for different sources of shocks, we experiment with different combinations of industry, country and time fixed effects, as discussed below.[12]

Table 2 presents our main set of results. Our benchmark measure of inequality is the log of the 90-10 ratio of the wage bill per worker across firms at the industry-country-year level. Concentration is measured based on the distribution of firm sales in the first three panels, and based on the distribution of employment in the bottom panel. Each panel uses a different measure of concentration, and each column considers a specification with a different set of fixed effects, as detailed in the bottom panel of the table. All regressions are weighted using each industry's time-averaged share of total value added in each country. Using time-averaged industry shares allows us to rule out any effects coming from changes in the industry structure within countries, while giving equal total weight to each country-year cell.

In the top panel of Table 2, our concentration measure is based on the sales of the top 1% of firms. Column (1) presents a specification which includes industry, country and year fixed effects. We find that there is a positive and statistically significant correlation between concentration and between-firm wage inequality. The remaining columns of Table 2 consider different combinations of fixed effects in order to exploit different sources of identification. Column (2) includes country-year and industry fixed effects. This would control for any country-specific policy changes that affect outcomes across industries. Identification is achieved from differential variation across industries within country-year cells. Column (3) includes industry-country and year fixed effects. This would account for any country-specific differences in industry-level outcomes. Here, identification is achieved from differential changes over time for a given industry within

[12] The inclusion of these different sets of fixed effects also alleviates potential concerns about the cross-country comparability of the data sources underlying CompNet.

a given country. Columns (4) through (6) include different combinations of two-way fixed effects. Regardless of the source of variation used for identification, the positive correlation between concentration and inequality remains. Even in the most restrictive specifications in Columns (4) through (6), the coefficients remain statistically significant at the 5% level or higher.

The second and third panels of Table 2 verify the robustness of our main result using alternative measures of sales concentration. In the second panel, concentration is measured as the share of sales of the top 10 firms in an industry-country-year cell. In all specifications, the estimated coefficient is positive, and statistically significant at the 5% level or higher. The third panel of Table 2 uses the Herfindahl-Hirschman index of market concentration. Once again, we find that the estimated correlations are always positive and statistically significant at the 10% level or higher.

In the final panel of Table 2, we replace our sales-based measures of concentration with a measure of employment concentration. This is analogous to the sales concentration measure based on the top 1% of firms, but now using the relative size (in terms of number of workers) for firms in the top 1% of the size distribution within an industry-country-year cell. The results once again suggest a positive correlation between concentration and inequality.

Appendix Table A.1 further explores the relationship between concentration and inequality using the standard deviation of the wage bill per worker across firms as an alternative measure of between-firm wage inequality.[13] We also explore inequality in the top half and the bottom half of the distribution, respectively, by focusing on the log 90-50, and the log 50-10 ratio of firm-level wages, as well as wage inequality between the very top of the distribution (99th percentile) and the 10th percentile. The results show that the relationship between concentration and wage inequality is generally observed throughout the entire distribution, with the estimated coefficients being positive and statistically significant in almost all cases. The magnitude of the estimated correlation tends to be larger when we focus on the very top of the distribution.

In order to delve deeper into this pattern, Figure 2 analyzes the link between concentration and wages throughout the full distribution of firm-level wages. In particular, we regress firm-level wages at percentile p in an industry-country-year cell on concentration in that cell, as well as industry-by-country and time fixed effects. The figure plots the estimated coefficients and 95% confidence intervals obtained from these regressions

[13]We exclude the top 1% of observations in terms of the standard deviation of the wage bill as they are extreme outliers.

at different percentile levels p, ranging from the first to the 99th percentile. Consistent with our finding regarding the positive correlation between concentration and inequality, we find that concentration is associated with a widening of the firm-wage distribution. Wages at the bottom of the distribution are lower in more concentrated industries, while wages at the top are higher. The higher wages at the top are particularly prominent when focusing on the 99th percentile, once again suggesting that the very top firms play a role in driving the increase in inequality, though even if the top 1% is excluded, inequality would still be higher in more concentrated industries.

Next, we explore the extent to which our results hold within each of the countries in our sample. To do so, we run a series of separate regressions of inequality on concentration for each country, including industry and time fixed effects. Panel A of Figure 3 plots the estimated coefficient on concentration for each country, along with the corresponding 95% confidence interval. The correlation between concentration and inequality (conditional on industry and time fixed effects) is positive in nine out of the 12 countries in our sample, and is statistically significant at the 1% level for six of these nine countries. This confirms that the results are widespread across European countries and do not seem to be particularly related to country-specific institutions.

Finally, Panel B of Figure 3 explores the extent to which our results hold within each of the broad sectors in our sample. In an analogous way to our country-specific analysis, we run a series of separate regressions of inequality on concentration for each broad sector, now including country and time fixed effects. We find that the correlation between concentration and inequality (conditional on country and time fixed effects) is positive and statistically significant in all but two of the broad sectors. The only exceptions are the real estate sector, where the estimated correlation is negative (though noisy), and the administrative support service sector, where the estimated correlation is estimated to be close to zero. Overall, we conclude that the positive association between concentration and inequality is widespread across different sectors of the economy.

5 Conclusions

We document a theoretical and an empirical link between rising concentration and rising between-firm wage inequality. Conceptually, a shock that favors the most productive firms in an industry (e.g. an increase in consumer price sensitivity, as in Autor et al. (2019)), will increase the concentration of employment and revenues in those firms. In a setting that links firm demand to worker wages (e.g. the search and bargaining framework of Helpman et al. (2010) or the fair wage framework of Egger and Kreickemeier (2012)), the expansion of production in the most productive firms will be accompanied by an increase in the relative wages of workers in those firms, hence increasing between-firm wage inequality.

We confirm the empirical relevance of this conceptual link using data on concentration and between-firm wage inequality for 14 European countries over the period 1999-2016. We indeed find evidence of a statistically significant positive correlation between concentration and inequality at the industry-country-year level, which is robust to allowing for a variety of different combinations of industry, country and year fixed effects. This positive correlation is also observed within most countries and sectors in Europe.

Further understanding the driving forces behind the rise in concentration and between-firm wage inequality, as well as the underlying micro-level adjustments occurring at the firm level using detailed micro-data would be promising avenues for future research.

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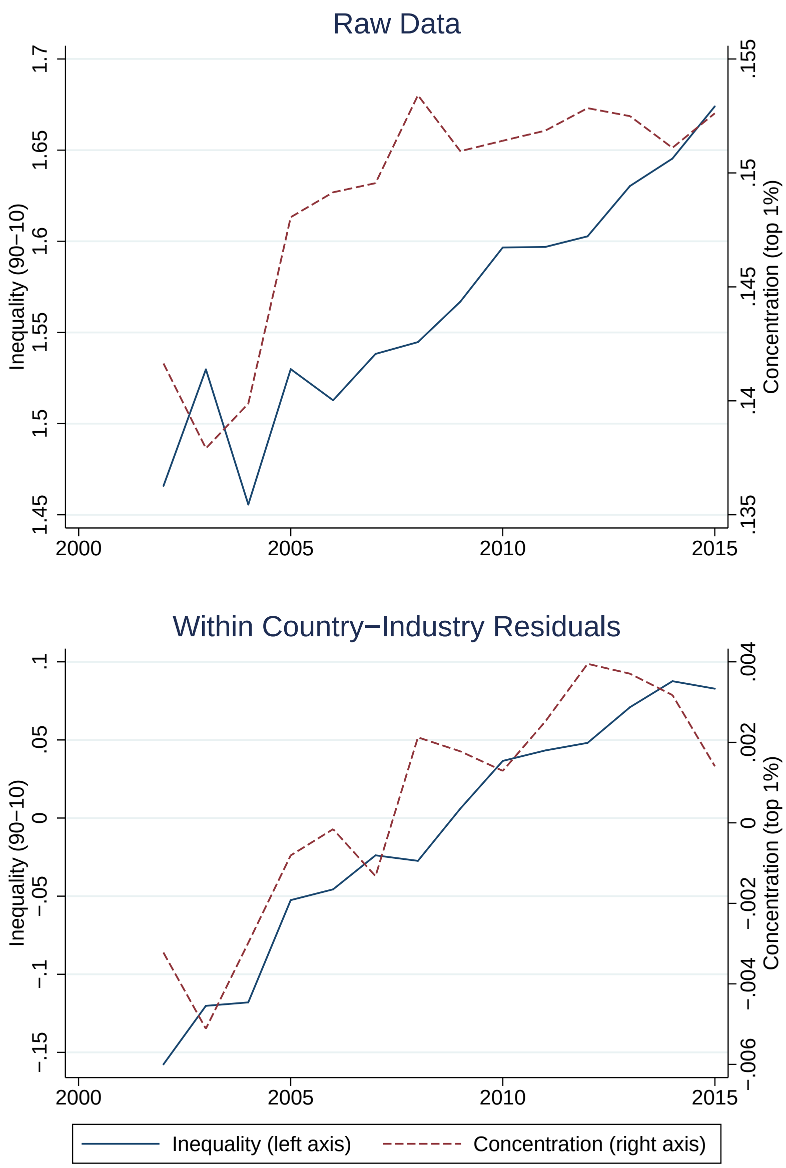
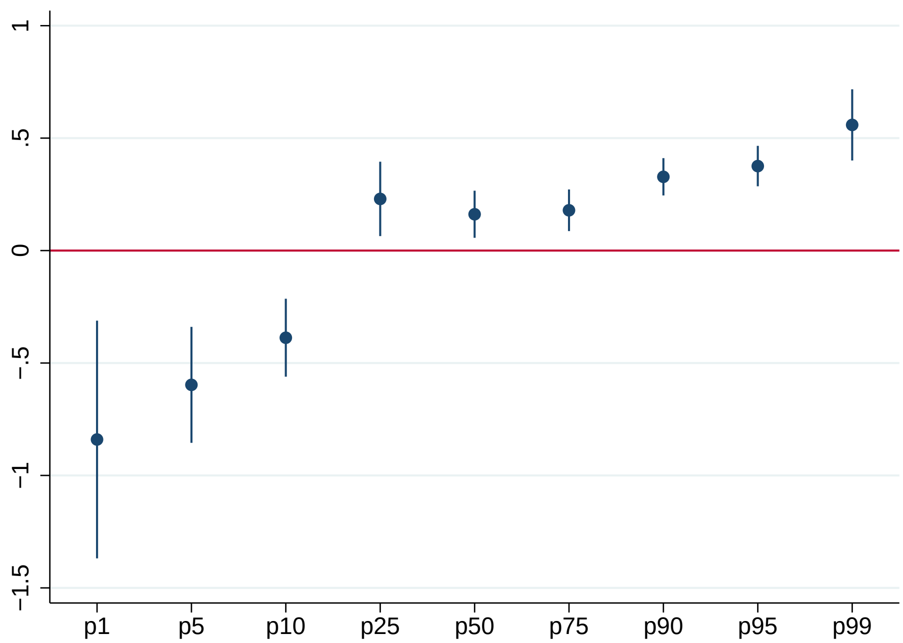
Figure 1: Rising Concentration and Rising Between-Firm Wage Inequality in Europe



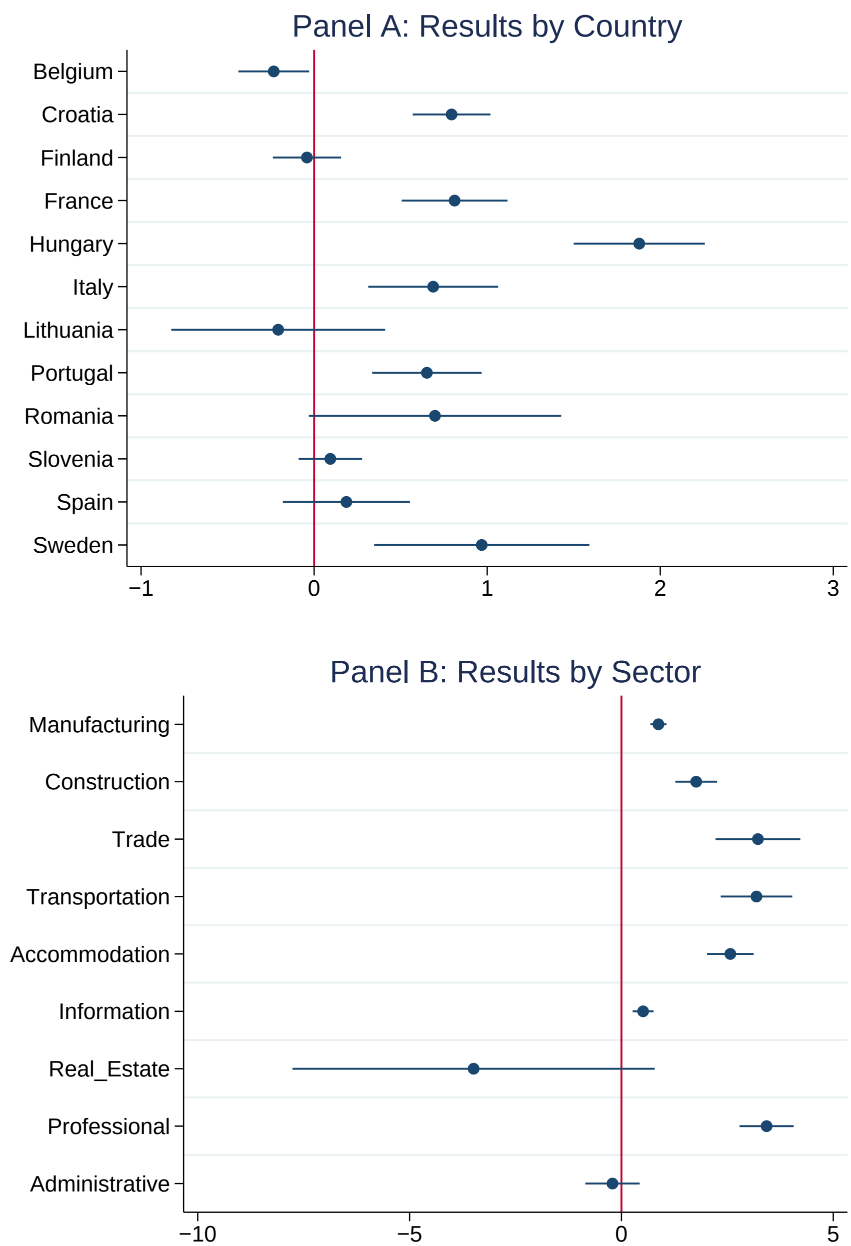
Note: Inequality measures are based on the 90-10 ratio of firm-level wages at the industry-country level. Concentration measures are based on the share of sales within the top 1% of firms in an industry-country cell. The top panel presents averages across up to 12 European countries based on the raw data, though the composition of countries varies over time given that not all countries are observed in all years (see Table 1). The bottom panel accounts for these changes in composition by presenting average residuals obtained from regressions of inequality and concentration on a full set of industry-country fixed effects, hence capturing only variation over time within industry-country cells.

Figure 2: Results across the Distribution of Firm-Level Wages



Note: The figure plots the estimated coefficients and 95% confidence intervals obtained from regressions of wages at different percentiles on concentration, controlling for industry-by-country and time fixed effects. Regressions are weighted using each industry's time-averaged share of total value added in each country. The concentration measure is based on the share of sales within the top 1% of firms in each country-industry-year cell.

Figure 3: Results by Country and by Broad Sector



Note: The figure plots the estimated coefficients and 95% confidence intervals obtained from regressions of inequality on concentration. In Panel A, regressions are run separately for each country, controlling for industry and time fixed effects. In Panel B, regressions are run separately for each broad sector, controlling for country and time fixed effects. All regressions are weighted using each industry's time-averaged share of total value added in each country. The concentration measure is based on the share of sales within the top 1% of firms in each country-industry-year cell. The inequality measure is based on the 90-10 ratio of firm-level wages in each country-industry-year cell.

Table 1: Change in inequality and concentration

Panel A: By Country

			Inequality			Concentration		
Country	$t=0$	$t=T$	$t=0$	$t=T$	Δ	$t=0$	$t=T$	Δ
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Belgium	2004	2015	1.03	1.14	0.11	12.60	12.25	-0.35
Croatia	2002	2016	1.37	1.58	0.21	15.45	16.40	0.96
Finland	1999	2015	1.11	1.03	-0.08	13.88	15.59	1.70
France	2004	2014	1.51	1.53	0.02	15.50	14.03	-1.47
Hungary	1999	2015	1.67	2.31	0.65	14.44	15.99	1.55
Italy	2001	2014	1.38	1.48	0.10	9.60	10.85	1.25
Lithuania	2000	2015	1.70	2.61	0.91	14.09	17.58	3.48
Portugal	2006	2015	1.21	1.53	0.32	14.44	14.84	0.40
Romania	2005	2015	2.27	2.06	-0.21	16.35	15.94	-0.41
Slovenia	2005	2016	0.98	0.91	-0.07	15.87	18.18	2.32
Spain	2009	2015	1.30	1.37	0.07	12.07	12.82	0.75
Sweden	2003	2015	1.59	2.10	0.51	12.32	13.84	1.53

Panel B: By Broad Sector

	Inequality			Concentration		
Broad Sector	2004	2014	Δ	2004	2014	Δ
	(3)	(4)	(5)	(6)	(7)	(8)
Manufacturing	1.29	1.48	0.19	16.71	17.59	0.88
Construction	1.26	1.53	0.27	12.01	12.59	0.58
Trade	1.42	1.69	0.27	13.88	14.77	0.89
Transportation	1.32	1.59	0.27	13.40	14.11	0.71
Accommodation	1.28	1.47	0.19	10.44	10.33	-0.11
Information	1.78	1.97	0.18	17.80	20.06	2.26
Real Estate	1.94	2.25	0.32	13.73	13.60	-0.13
Professional	1.73	1.93	0.21	12.57	12.02	-0.55
Administrative	1.61	1.95	0.34	14.10	14.80	0.70

Note: In Panel A, $t = 0$ ($t = T$) denotes the first (last) appearance of both inequality and concentration measures in the data for the respective country. Inequality measures are based on the 90-10 ratio of firm-level wages. Concentration measures are based on the share of sales within the top 1% of firms. Due to missing data, Denmark and Netherlands are excluded from Panel A, but they are included in regressions that use alternative measures of inequality and/or concentration. In order to compute the sector-level averages in Panel B, we first compute sector-level measures for each country, by averaging across 2-digit industries using industry shares of value added as weights. We then compute an average across countries in each year, giving equal weight to each country. The sample is restricted to the set of countries which have consistent information for all sectors over the 2004-2014 period, which are Belgium, Croatia, Finland, France, Hungary, Italy and Lithuania, except for the Real Estate sector which excludes Finland.

Table 2: Concentration and Wage Inequality

	Dep var: log 90-10 ratio of wage bill per worker					
	(1)	(2)	(3)	(4)	(5)	(6)
Concentration (top 1%)	0.373	0.322	0.743	0.198	0.639	0.645
	(0.078)***	(0.071)***	(0.081)***	(0.079)**	(0.095)***	(0.061)***
Obs.	7037	7032	7033	6980	6981	7027
R^2	0.835	0.870	0.927	0.881	0.938	0.961
Concentration (top 10)	0.237	0.197	0.197	0.176	0.136	0.179
	(0.036)***	(0.029)***	(0.054)***	(0.03)***	(0.064)**	(0.036)***
Obs.	8175	8168	8174	8158	8165	8168
R^2	0.689	0.814	0.811	0.842	0.845	0.922
Concentration (HHI)	0.183	0.231	0.154	0.503	0.286	0.33
	(0.072)**	(0.059)***	(0.083)*	(0.065)***	(0.103)***	(0.057)***
Obs.	8820	8813	8819	8798	8805	8813
R^2	0.683	0.798	0.817	0.833	0.847	0.918
Concentration (Employment)	0.291	0.160	0.699	0.014	0.718	0.417
	(0.084)***	(0.077)**	(0.088)***	(0.084)	(0.101)***	(0.067)***
Obs.	7176	7171	7174	7121	7124	7168
R^2	0.832	0.867	0.927	0.879	0.938	0.960
Industry FE	Yes	Yes				
Country FE	Yes					
Year FE	Yes		Yes			
Country x Year FE		Yes		Yes		Yes
Industry x Country FE			Yes		Yes	Yes
Industry x Year FE				Yes	Yes	

Note: Observations are at the country-industry-year level. All regressions are weighted using each industry's time-averaged share of total value added in each country. The concentration measures in the top three panels are based on the distribution of sales across firms within each country-industry-year cell.

Online Appendix for:

Rising Concentration and Wage Inequality

Guido Matias Cortes (York University)
Jeanne Tschopp (University of Bern)

Appendix A Deriving Concentration Measures and the Wage Distribution

Concentration Measures Let $\overline{\theta}$ denote the productivity level corresponding to the $(100-\mu)$th percentile of the productivity distribution. The share of sectoral revenues accruing to firms in the top μ% of the productivity distribution is given by:

$$\begin{aligned} C_r &= 1 - \frac{\int_{\theta_d}^{\overline{\theta}} r(\theta) dG_\theta(\theta)}{\int_{\theta_d}^{\infty} r(\theta) dG_\theta(\theta)} \\ &= 1 - \frac{\int_{\theta_d}^{\overline{\theta}} \theta^{\frac{\beta}{\Gamma}} dG_\theta(\theta)}{\int_{\theta_d}^{\infty} \theta^{\frac{\beta}{\Gamma}} dG_\theta(\theta)} \\ &= \left(\frac{\overline{\theta}}{\theta_d}\right)^{\frac{\beta}{\Gamma}-z}. \end{aligned} \tag{A.1}$$

where the second equality is obtained using the equation of equilibrium firm-level revenues $r(\theta) = r_d \left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\Gamma}}$, and the last equality uses the fact that with a Pareto distribution $g(\theta) = z\theta^{-(z+1)}$ and $1 - G_\theta(\theta_d) = \theta_d^{-z}$.

Finally, to express C_r as a function of μ, it is useful to relate $\frac{\overline{\theta}}{\theta_d}$ to the share of firms with $\theta \geq \overline{\theta}$:

$$\begin{aligned} \mu &= 1 - \int_{\theta_d}^{\overline{\theta}} g(\theta \mid \theta \geq \theta_d) d\theta \\ &= 1 - \frac{1}{\theta_d^{-z}} \int_{\theta_d}^{\overline{\theta}} z\theta^{-(z+1)} d\theta \\ &= \left(\frac{\overline{\theta}}{\theta_d}\right)^{-z}. \end{aligned}$$

It follows that $\frac{\bar{\theta}}{\theta_d} = \mu^{-\frac{1}{z}}$. Replacing the latter equation in (A.1) we obtain an expression for the share of revenues accruing to firms in the top $\mu\%$ of the productivity distribution:

$$C_r = \mu^{1-\frac{\beta}{\Gamma z}}. \tag{A.2}$$

The concentration measure of employment is obtained in a similar way by computing the share of sectoral employment concentrated in the firms in the top $\mu\%$ of the productivity distribution.

Wage Distribution Let θ_w denote the productivity level associated with $w(\theta_w) = w$. The wage distribution is given by:

$$\begin{aligned} G_f(w) &= Pr\left[w(\theta) \leq w\right] \\ &= Pr\left[\theta \leq \theta_w \mid \theta \geq \theta_d\right] \\ &= 1 - \left(\frac{\theta_d}{\theta_w}\right)^z, \end{aligned} \tag{A.3}$$

where the last equality uses the fact that productivity follows a Pareto distribution with shape parameter z. Finally, using the fact that $w(\theta_w) = w_d\left(\frac{\theta_w}{\theta_d}\right)^{\frac{\beta k}{\delta\Gamma}}$, we have that $\frac{\theta_d}{\theta_w} = \left(\frac{w_d}{w}\right)^{\frac{\delta\Gamma}{\beta k}}$ and we can rewrite (A.3) as follows:

$$G_f(w) = 1 - \left(\frac{w_d}{w}\right)^{\frac{\delta\Gamma z}{\beta k}}. \tag{A.4}$$

Hence, firm-level wages are Pareto distributed with scale parameter w_d and shape parameter $\frac{\delta\Gamma z}{\beta k}$.

Appendix B Productivity Threshold for Production and Selection

In the closed economy version of the Helpman et al. (2010) model, the equilibrium productivity cutoff for production is given by:

$$\theta_d = \left(\frac{\beta}{z\Gamma - \beta}\right)^{1/z}\left(\frac{f_d}{f_e}\right)^{1/z}\theta_{min}. \tag{A.5}$$

From (A.5) it is straightforward to see that

$$\frac{\partial \theta_d}{\partial \beta} = \left(\frac{f_d}{f_e}\right)^{1/z} \left(\frac{\beta}{z\Gamma - \beta}\right)^{1/z-1} \frac{\theta_{min}}{(z\Gamma - \beta)^2} > 0. \tag{A.6}$$

Hence, an increase in the elasticity of substitution increases the productivity threshold for production and leads to a reduction in the range of firm types.

Given that productivity follows a Pareto distribution, the variance of productivity among operating firms is given by

$$\frac{z\theta_d^2}{(z-1)^2(z-2)},$$

which implies that an increase in θ_d will increase the variance of productivity among operating firms. Intuitively, this is due to the exit of firms with relatively homogeneous firm types at the bottom of the productivity distribution and to the relative increase in the mass of firms towards the tail.

Appendix C Concentration, Wage Inequality and Worker Sorting

Helpman et al. (2010, Section 5.1) present an extension to their model that allows for worker heterogeneity in observable characteristics. This makes it possible to also think about the impacts operating through the increased sorting of good workers to good firms in terms of observables. Here we illustrate the key features of this extension to the model, and derive the key predictions of interest for our purposes regarding concentration and between-firm wage inequality.

Consider an economy with two types of workers, $\ell = H, L$, with H denoting skilled workers and L unskilled workers. The production function is given by:

$$y = \theta \left(\bar{a}_H h_H^{\gamma_H}\right)^{\lambda_H} \left(\bar{a}_L h_L^{\gamma_L}\right)^{\lambda_L}, \qquad \lambda_H + \lambda_L = 1 \tag{A.7}$$

The match-specific ability of each group has a Pareto distribution with shape parameter k_ℓ and lower bound $a_{min,\ell}$. Search and matching for skilled and unskilled workers occur in separate markets, so search costs b_ℓ are allowed to differ by type.

Helpman et al. (2010) show that firm-level employment and wages for workers of type ℓ are given by:

$$h_\ell(\theta) = h_{d,\ell}\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\Gamma}(1-k_\ell/\delta)}, \qquad h_{d,\ell} \equiv \frac{\lambda_\ell \beta \gamma_\ell}{\Gamma}\frac{f_d}{b_\ell}\left[\frac{\lambda_\ell \beta(1-\gamma_\ell k_\ell)}{\Gamma}\frac{f_d}{ca^{\delta}_{min,\ell}}\right]^{-k_\ell/\delta}$$

$$w_\ell(\theta) = w_{d,\ell}\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta k_\ell}{\delta\Gamma}}, \qquad w_{d,\ell} \equiv b_\ell\left[\frac{\lambda_\ell \beta(1-\gamma_\ell k_\ell)}{\Gamma}\frac{f_d}{ca^{\delta}_{min,\ell}}\right]^{k_\ell/\delta}$$

where now:

$$\Gamma = 1 - \beta(\lambda_H\gamma_H + \lambda_L\gamma_L) - \frac{\beta}{\delta}[1 - (\lambda_H\gamma_H k_H + \lambda_L\gamma_L k_L)]$$

The relative employment of skilled workers within a firm with productivity θ is given by:

$$\frac{h_H(\theta)}{h_L(\theta)} = \frac{h_{d,H}}{h_{d,L}}\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\delta\Gamma}(k_L-k_H)}$$

And the relative wage of skilled workers is given by:

$$\frac{w_H(\theta)}{w_L(\theta)} = \frac{w_{d,H}}{w_{d,L}}\left(\frac{\theta}{\theta_d}\right)^{\frac{\beta}{\delta\Gamma}(k_H-k_L)}$$

For sufficiently high values of $\frac{w_{d,H}}{w_{d,L}}$, we have that in all firms, skilled workers are paid more than unskilled workers, i.e. $\frac{w_H(\theta)}{w_L(\theta)} > 1\ \forall\theta$.

Assuming that $k_H < k_L$, i.e. that the match-specific ability distribution is more dispersed among skilled workers than among unskilled workers, we have that the relative employment of skilled workers is increasing in firm productivity.

Average firm wages will be higher in more productive firms because they: (i) employ a larger proportion of skilled workers, and (ii) pay higher wages to both worker types. Hence, wages differ across firms both because of the composition/sorting of workers, and because of firm premia conditional on worker type.

The concentration of type ℓ workers in the top μ% of firms is given by:

$$C_{h,\ell} = \mu^{1-\frac{\beta}{\Gamma_z}(1-k_\ell/\delta)} \tag{A.8}$$

Given that $k_H < k_L$, we have that $C_{H,h} > C_{L,h}$.

We have the following prediction:

Prediction: An increase in the elasticity of substitution, σ, increases concentration of employment in the most productive firms, particularly so for skilled workers:

$$\frac{\partial C_{h,H}}{\partial \sigma} > \frac{\partial C_{h,L}}{\partial \sigma} > 0$$

Corollary: The disproportionate increase in employment concentration for skilled workers implies stronger sorting of skilled workers to high productivity firms. This increased sorting and the implied changes in the composition of workers across firm types will increase between-firm inequality in average firm-level wages.

The distribution of wages across firms for workers of type ℓ is given by:

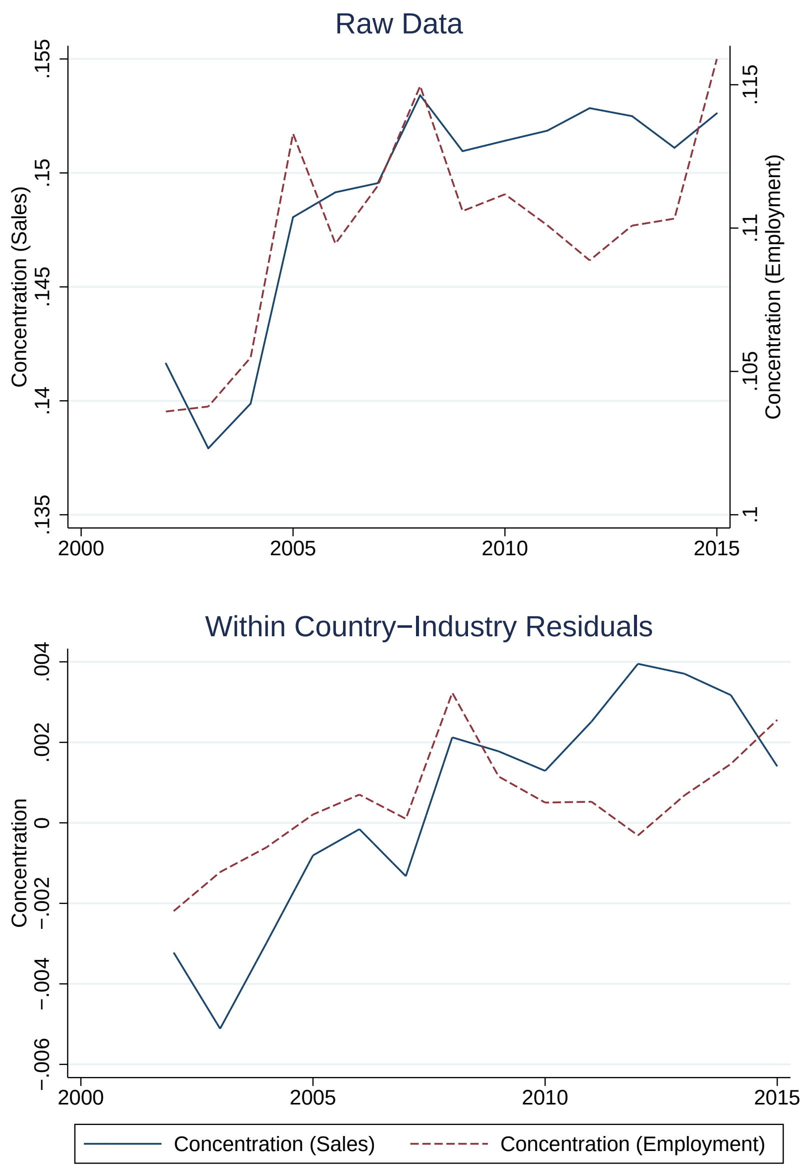
$$G_f(w_\ell) = 1 - \left(\frac{w_{d,\ell}}{w_\ell}\right)^{\frac{\delta \Gamma z}{\beta k_\ell}}$$

This is a Pareto distribution with scale parameter $w_{d,\ell}$ and shape parameter $\frac{\delta \Gamma z}{\beta k_\ell}$. Inequality, as measured by any scale-invariant measure, will be a function of the shape parameter only.

Prediction: An increase in the elasticity of substitution, σ, increases within-group, between-firm wage inequality for both worker types.

Corollary: An increase in the elasticity of substitution, σ, increases inequality in average firm wages both because of (i) increased worker sorting and (ii) increased dispersion in firm premia conditional on worker types.

Figure A.1: Rising Concentration: Sales vs Employment



Note: Concentration measures are based on the share of turnover within the top 1% of firms, and the share of employment within the top 1% of firms in an industry-country cell, respectively. The top panel presents averages across up to 12 European countries based on the raw data, though the composition of countries varies over time given that not all countries are observed in all years. The bottom panel accounts for these changes in composition by presenting average residuals obtained from regressions of each concentration measure on a full set of industry-country fixed effects, hence capturing only variation over time within industry-country cells.

Table A.1: Alternative Measures of Wage Inequality

	(1)	(2)	(3)	(4)	(5)	(6)
	Dep var: Standard Deviation of wage bill per worker					
Concentration (top 1%)	10.308	12.379	1.785	12.884	-3.991	5.951
	(3.014)***	(2.865)***	(4.053)	(3.088)***	(4.714)	(3.786)
Obs.	7780	7780	7776	7731	7726	7776
R^2	0.54	0.603	0.662	0.656	0.715	0.725
	Dep var: log 90-50 ratio of wage bill per worker					
Concentration (top 1%)	0.415	0.443	0.166	0.435	0.089	0.243
	(0.055)***	(0.055)***	(0.040)***	(0.061)***	(0.046)*	(0.035)***
Obs.	7635	7635	7629	7591	7584	7629
R^2	0.547	0.575	0.899	0.598	0.917	0.927
	Dep var: log 50-10 ratio of wage bill per worker					
Concentration (top 1%)	0.246	0.169	0.560	0.100	0.591	0.374
	(0.062)***	(0.053)***	(0.072)***	(0.059)*	(0.086)***	(0.052)***
Obs.	7037	7032	7033	6980	6981	7027
R^2	0.805	0.863	0.892	0.878	0.906	0.948
	Dep var: log 99-10 ratio of wage bill per worker					
Concentration (top 1%)	0.376	0.351	1.020	0.118	0.882	0.956
	(0.101)***	(0.095)***	(0.112)***	(0.103)	(0.127)***	(0.096)***
Obs.	6954	6949	6950	6894	6895	6944
R^2	0.82	0.849	0.91	0.871	0.929	0.938
Industry FE	Yes	Yes				
Country FE	Yes					
Year FE	Yes		Yes			
Country x Year FE		Yes		Yes		Yes
Industry x Country FE			Yes		Yes	Yes
Industry x Year FE				Yes	Yes	

Note: Observations are at the country-industry-year level. All regressions are weighted using each industry's time-averaged share of total value added in each country. The concentration measures are based on the distribution of sales across firms within each country-industry-year cell.